Techniques of Calculus

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These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

INTRODUCTION

It is assumed that the reader of these notes has acquired the basic concepts of one variable calculus, can differentiate and integrate polynomials and has seen some of the applications. This material is covered in my notes *Elementary Calculus* or *Concepts of Calculus*.

These notes are intended to extend the range of functions that can be differentiated, introducing the exponential, logarithmic and trigonometric functions. It covers certain techniques such as the Mean Value Theorem and L'Hôpital's Rule.

Integration can be defined as anti-differentiation, that is, the integral of a function f(x) is that function, up to an arbitrary constant, that gives f(x) when differentiated. The weakness of this is that unless you already know the value of the integral you cannot know that the integral exists. Since many important functions can only be defined as an integral, this is unsatisfactory. The Riemann integral defines integrability independently of differentiation. Then the Fundamental Theorem of Calculus is proved whereby integration is shown to be the reverse process to differentiation.

Many integrals cannot be expressed in terms of the elementary functions – polynomials, exponential, logarithmic and trigonometric functions. Many others that can be so expressed require special techniques for evaluating. The chapter on techniques of integration discusses many such methods.

Then follow chapters on the convergence or divergence of infinite series and the calculus of several (mainly two) variables. Finally there are chapters on the differential calculus of functions of several variables and an introduction to ordinary differential equations.

Two additional items that weren’t in the previous edition and which, to my knowledge, can’t be found anywhere else, are integration closed spaces of functions and an extension of Newton’s Method to systems of two equations in two variables. The latter provides a way of obtaining non-real zeros to functions of a complex variable.

The notes are still incomplete. In a later edition I intend to include exercises and solutions for every chapter. Also chapters on Multiple Integrals and Vector Calculus will be added.

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