

These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

INTRODUCTION

It is assumed that the reader of these notes has acquired the basic concepts of one variable calculus, can differentiate and integrate polynomials and has seen some of the applications. This material is covered in my notes *Elementary Calculus* or *Concepts of Calculus*.

These notes are intended to extend the range of functions that can be differentiated, introducing the exponential, logarithmic and trigonometric functions. It covers certain techniques such as the Mean Value Theorem and L'Hôpital's Rule.

Integration can be defined as anti-differentiation, that is, the integral of a function $f(x)$ is that function, up to an arbitrary constant, that gives $f(x)$ when differentiated. The weakness of this is that unless you already know the value of the integral you cannot know that the integral exists. Since many important functions can only be defined as an integral, this is unsatisfactory. The Riemann integral defines integrability independently of differentiation. Then the Fundamental Theorem of Calculus is proved whereby integration is shown to be the reverse process to differentiation.

Many integrals cannot be expressed in terms of the elementary functions – polynomials, exponential, logarithmic and trigonometric functions. Many others that can be so expressed require special techniques for evaluating. The chapter on techniques of integration discusses many such methods.

Then follow chapters on the convergence or divergence of infinite series and the calculus of several (mainly two) variables. Finally there are chapters on the differential calculus of functions of several variables and an introduction to ordinary differential equations.

Two additional items that weren't in the previous edition and which, to my knowledge, can't be found anywhere else, are integration closed spaces of functions and an extension of Newton's Method to systems of two equations in two variables. The latter provides a way of obtaining non-real zeros to functions of a complex variable.

The notes are still incomplete. In a later edition I intend to include exercises and solutions for every chapter. Also chapters on Multiple Integrals and Vector Calculus will be added.

CONTENTS

1. DIFFERENTIATION

1.1 The Real Number System	11
1.2 Absolute Values	15
1.3 Completeness	17
1.4 Limits	19
1.5 Continuous Functions	25
1.6 Differentiation Theorems	31
1.7 Well-Known Derivatives	37
1.8 L'Hôpital's Rule	45
1.9 The Mean Value Theorem	48
1.10 Maxima And Minima	51
1.11 The Second Derivative Test	54
1.12 The First Derivative Test	58
1.13 Which Test Should You Use?	62
1.14 Global Maximums And Minimums	65
1.15 Newton's Method	70
1.16 The Newton Spreadsheet	73
1.17 Where Do We Start?	76
Exercises For Chapter 1	80
Solutions For Chapter 1	82

2. INTEGRATION

2.1 A Review of Integral Calculus	87
2.2 The Riemann Integral	88
2.3 The Fundamental Theorem of Calculus	96
2.4 Areas Between Curves	98
2.5 What If Curves Cross	104
2.6 Volumes of Revolution	109
2.7 Numerical Integration	111
2.8 The Cubic Fit Method	117
Exercises For Chapter 2	126
Solutions For Chapter 2	130

3. TECHNIQUES OF INTEGRATION

3.1 Integrating in Terms of Elementary Functions	151
3.2 Integration by Substitution	152
3.3 Integration by Parts	158
3.4 Trigonometric Substitution	166
3.5 Partial Fractions	170
3.6 Reduction Formulae	179
3.7 The t -Method	185
3.8 Length of Curves	186
3.9 Parametric Representation of Curves	194
3.10 Integral Closed Spaces	203

4. SEQUENCES AND SERIES

4.1 Limits of Sequences	209
4.2 Infinite Series	215
4.3 The Harmonic Series	218
4.4 Series With Positive Terms	224
4.5 The p -Series	230
4.6 Alternating Series	232
4.7 Absolute Convergence	235
4.8 Power Series	241
4.9 Differentiability of Power Series	247
4.10 Taylor Series And Maclaurin Series	248

5. PARTIAL DERIVATIVES

5.1 Functions Of Several Variables	251
5.2 Partial Derivatives	254
5.3 Higher Partial Derivatives	256
5.4 The Tangent Plane and the Normal to a Surface	258
5.5 Implicit Form for Surfaces	260
5.6 Maxima and Minima for Functions of Two Variables	262
5.7 The Nature of Stationary Points for Functions of Two Variables	263
5.8 Lagrangian Points	265
5.9 Newton's Method For Two Variables	271
5.10 Newton's Method For Complex Zeros	276
5.11 Newton's Method For Stationary Points ...	278

6. FIRST ORDER ORDINARY ODEs	
6.1 Introduction	283
6.2 Mathematical Models	286
6.3 ODEs That Can Be Solved By Direct Integration	293
6.4 Separable ODEs	295
6.5 Homogeneous ODEs	296
6.6 Linear ODEs	298
6.7 Exact ODEs	301
6.8 Initial Conditions	303
7. LINEAR ODEs WITH CONSTANT COEFFICIENTS	
7.1 Higher Order ODEs	311
7.2 Linear ODEs With Constant Coefficients ...	314
7.3 The Homogeneous Case	315
7.4 Finding Particular Solutions	319
8. MULTIPLE INTEGRALS	
8.1 Double Integrals Over a Rectangle	329
8.2 Iterated Integrals	331
8.3 Simpson's Rule For 2 Variables	334
8.4 Proof Of Theorem 3	339
8.5 Integrals Over Other Regions	342
8.6 Centre Of Mass In 2 Dimensions	345
Exercises For Chapter 8	351
Solutions For Chapter 8	352

9. VECTOR CALCULUS	
9.1 A Revision Of Vector Algebra	355
9.2 Basic Calculus Of Vector Functions	362
9.3 The Gravitational Field	366
9.4 Work Done In A Force Field	368
9.5 Line Integrals	372
9.6 The Fundamental Theorem of Line Integrals	374
9.7 Conservative Vector Fields	374
APPENDIX A: Partial Fractions	377
APPENDIX B: Generalised Simpson's Rule ...	381

